Application of the TLM-Method for the Analysis of Longitudinally Periodic Structures

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Abstract:
The paper delivers new aspects to the TLM-method applied to longitudinally periodic structures. This contribution explains the main features of the method like the parameter estimation and the phase walls derived from the Floquet's theorem as well as examples and compares the obtained results to analytical solutions and measurements.

1 INTRODUCTION

In literature, the calculation of longitudinally periodic waveguides is usually performed by a simple approach. According to [1] the Floquet's theorem means that each field component $F(z)$ with $F \in \{E_{x,y,z}, H_{x,y,z}\}$ is dependent in the following form

$$F(z) = e^{\pm \gamma z}\, \Phi(z) \text{ with } \Phi(z+l) = \Phi(z), \quad (1)$$

whereby γ is the (usually complex) propagation constant and l the periodicity. This yields

$$F(z+l) = e^{\pm \gamma z} F(z). \quad (2)$$

A similar statement is given in [2], where only an imaginary value $j\beta$ is treated instead of a complex γ, and the used technique is the FDTD method. Other attempts treat the problem of periodic waveguides - also by the help of the Floquet's theorem - in the frequency-domain [3,4,6].

2 THEORETICAL BACKGROUND

In the following, we apply the Floquet's theorem to the TLM-method in combination with a new analytic method in order to reduce the simulation effort.

It is rather easy to show for periodically connected longitudinally homogeneous transmission lines that equations (1) and (2) are valid under the assumption of missing mode conversion. In [1] a periodic and capacitively loaded rectangular wave-guide is calculated. The validity of this approach is plausible because of obvious considerations, but generally it is difficult to prove it analytically.

The most general derivation of a system of differential equations with periodic coefficients as assumed by Floquet's theorem was found in [5]. The applied method is not represented here for complexity reasons, but nevertheless it has to be noticed the real part of the complex propagation coefficient $\gamma = \alpha + j\beta$ does not vanish. This is valid even for periodic waveguides with lossless materials. Nevertheless, for the simulation the approximation $\alpha \approx 0$ has to be used. Attenuations would hence become noticeable by a temporal reduction of the field energy similar to the case of a simulation of longitudinally homogeneous lossy lines. The approximation can be regarded as valid if the attenuations are not too great.

The simulation of longitudinally homogeneous and longitudinally periodic lines is consequently based

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on the same assumption of a coupling between the field components of the cross-sectional planes at z_1 and z_2. Hence, the same exchange algorithm can be used. This is however only under the assumption of two complete TLM-networks for the simulation valid. To reduce the effort to one mesh the knowledge about the z-dependency can be used. Considering

$$\cos(\xi + \Delta\xi) = v_1 \cos(\xi), \qquad (3)$$

with given factor v_1, $|v_1| \le 1$, $\xi = z_1$ and $\Delta\xi = \beta l$, $\Delta\xi$ can be determined as

$$\Delta\xi = \arccos\left(v_1 \cos(\xi)\right) - \xi. \qquad (4)$$

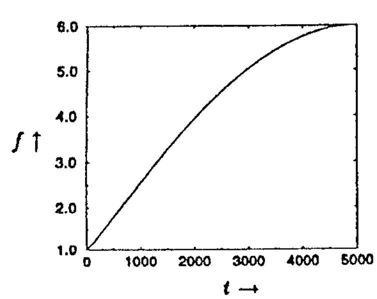
This expression should not depend on ξ because of having prescribed that $\Delta\xi = \beta l$. Thus, a simulation requires a fixed value of the zero phase i.e. at z_1.

This could be done simplest by the help of an electric ($\beta z_1 = \pi$) or magnetic ($\beta z_1 = 0$) wall near to z_1. Using a magnetic wall the coupling factor simplifies then to $v_1 = \cos(\beta l)$.

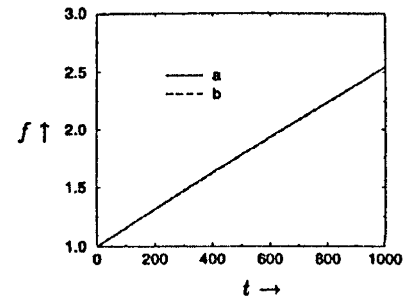
The formulation of the exchange algorithm can now be described easily. The magnetic wall be situated at z_1. Because of this magnetic wall in an SCN [9] for $U_2^h(z_1)$ yields: $U_2^h(z_1) = U_2^r(z_1)$. The quantity $U_9^r(z_2)$ is given by the scattering. Therefore, it has to be calculated in a way that $U_9(z_2) = v_1 U_2(z_1)$ is valid. Hence follows:

$$U_2^h(z_2) = 2 v_1 U_2^r(z_1) - U_9^r(z_2) \qquad (5)$$

and analogously for the gates 4 and 8.



a)



b)

Fig. 1: Sinusoidal function with dc-component: Test functions for automatic parameter analysis. t means the number of the function value. Diagram a): A quarter period of a sine function, Diagram b): Curve a is identical to the first fifth part of diagram a. Curve b is a straight line.

With this exchange algorithm for the cross-sectional planes at z_1 and z_2 some experiments have been successfully realized. Hereby, it was not necessary to be in particular carefully while exciting. This matches even with a statement given in [2], where also a magnetic wall made the algorithm "robust".

3 SIMULATIONS AND EVALUATIONS

We applied a method of parameter estimation already published in [8] to reduce the simulation time necessary for the analysis of basic modes. An optimizer searches for the parameters of a function given by

$$f(t) = a \sin\left(2\pi \frac{t}{T} + \varphi\right) e^{-\alpha t} + d \qquad (6)$$

to fit the analytic curve to the simulation data. This search starts using estimated values for the parameters having an essential influence on the accuracy of the results. To improve the accuracy we developed a simple program that estimates the parameters analytically. We applied both programs to the data demonstrated in Fig. 1. Even the parameters of the curve shown in Fig. 1b) were determined with a relative error of less than $1 * 10^{-4}$.

Hence it follows the possibility of reducing time consuming simulations of waveguide structures drastically compared to the analysis of parameters using Fourier transformations because the last ones require several complete periods in order to yield to some extent exact results.

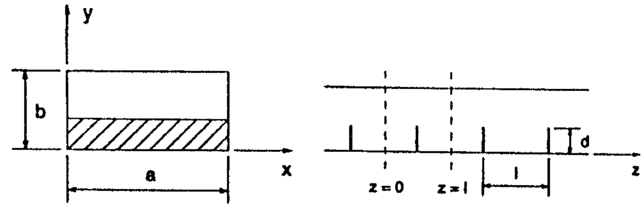


Fig. 2: Periodic and capacitively loaded ideal rectangular waveguide: $a = 20\text{mm}$, $b = 10\text{mm}$, $d = 3\text{mm}$, $l = 30\text{mm}$.

In [1] a periodic capacitively loaded rectangular waveguide is calculated. The arrangement is shown in Fig. 2. In this simulation the conventional excitation using a delta-impulse and analysis with the help of a Fourier transformation was performed. The frequencies of the simulation match well with the behaviour from the approximate calculations.

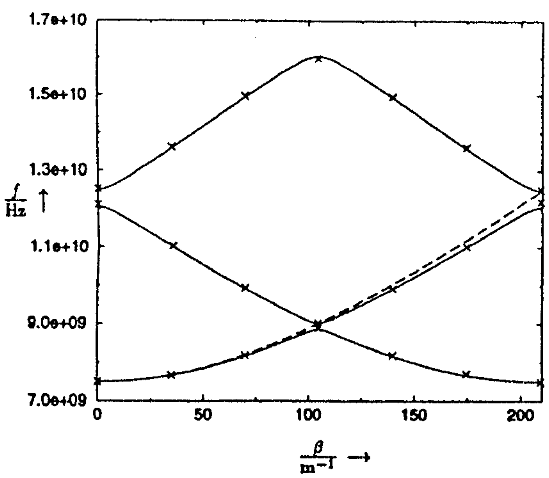


Fig. 3: f-β-diagram of a periodic capacitively loaded rectangular waveguide. Solid line: analytical approximative calculation according to [1]. Symbols: simulation results. Dashed line: curve of the H_{10}-type of the not capacitively loaded rectangular waveguide.

Next, a coplanar line with periodically loaded outer conductors is investigated (see Fig. 4). Perfectly conducting metal was assumed instead of gold for the conductors.

The excitation of the fundamental mode should result with the static electric field of the type. Therefore, the program using the method of static finite differences (SFD) has been applied to calculate and impress this field upon the TLM-mesh. Hence, the assumptions have been given for the dispersion analysis by the help of the parameter estimation of harmonic functions. The simulation time for each β-value was set to approximately half of a period duration.

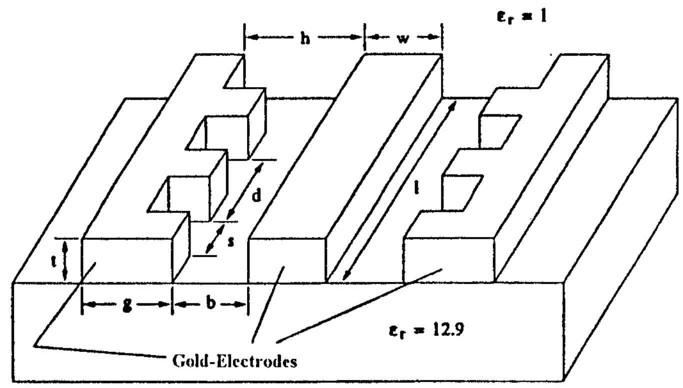


Fig. 4: Coplanar slow wave line according to [8]. Substrate: GaAs, geometrical data: $b = 9\mu m$, $d = 50\mu m$, $g = 200\mu m$, $h = 80\mu m$, $\omega = 28\mu m$, $s = 25\mu m$, $t = 1\mu m$, $l = 1cm$ (200 elementary cells with the length $d = 50\mu m$ of each for the measurement of the line).

The measurement results from [7] and the simulation results are represented in Fig. 5. The agreement between simulation and measurement is rather good.

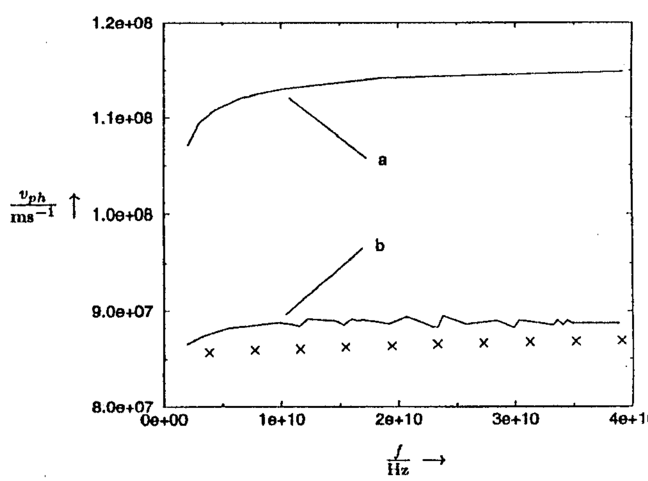


Fig. 5: The v_{ph}-f-diagram of a coplanar waveguide depicted in Fig. 4. a, b: measurement from [7]; x: simulation results; a: homogeneous line; b: periodically loaded line.

A first simulation with a simulation time of just a quarter period led to oscillating values of v_{ph}-phase. An analysis of a field component over the simulation time brought out several subharmonics which disturbed the above mentioned algorithms

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for the automatic parameter extraction. We were able to show that these higher-frequency harmonics appear immediately when the simulation parameter β is changed. Thus, we increased the simulation time to average out the subharmonics.

Another problem to be considered concerns the static field for the first excitation. In Fig. 6 two time intervals of the TLM-simulation are shown after excitation with the SFD-field.

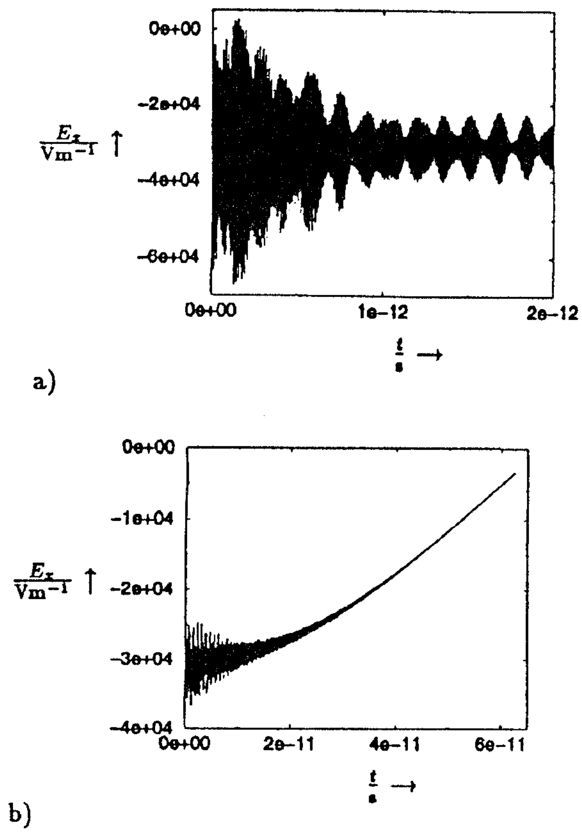

Fig. 6: Transients in the simulation of the coplanar line. Diagram a): 7,500 time steps after excitation with the static electric field of the fundamental mode at $\beta = 0\text{m}^{-1}$. Diagram b): 229,000 time steps in addition to diagram a) at $\beta = 282\text{m}^{-1}$.

Obviously, such noisy data disturb the parameter estimation which makes the relevance of an extensively terminated transient clear.

4 CONCLUSION

In the frame of this work diverse problems have been treated in order to be able to simulate and analyse longitudinally periodic structures efficiently using the TLM-method: Calculation of static electric fields by the help of the method of static finite differences, parameter estimation of harmonic functions and phase walls.

In conclusion it can be mentioned that it is possible to receive exact results for longitudinally periodic structures by applying the TLM-method combined with the phase walls described above.

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